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HYBRID METHODS FOR TWO-DIMENSIONAL CUTTING STOCK AND BIN PACKING PROBLEMS

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In a two-dimensional cutting stock (*2DCS*) problem it is intended to cut a set of rectangular items from a set of rectangular plates in such a way that the number of used plates is minimized. The plates are available in (a virtually) infinite number and all have the same dimensions, i.e., the same width and height.

A set of items to be cut, grouped by types according to their dimensions (width and height), is given. Each item type is defined by a width, a height and a demand (corresponding to the number of items of the type to be cut). This problem is equivalent to the two-dimensional bin packing problem, in which it is intended to place the set of items in the plate (bin) with the same goal.

The 2DCS problem and its variants arise in different industrial processes, such as the textile, the leather, the paper, the furniture and the sheet metal. This problem has been extensively studied in the last fifty years, due to its wide industrial applicability, however there are not general and efficient methods to solve it due to its computational complexity (NP-hard).

The motivation behind this work lies in the woodcut industry where plates of wood must be cut in pieces (items) to satisfy customer orders. In this industry, due to technological constraints, usually only cuts parallel to the sides of the plate (orthogonal cuts) and from one border to the opposite one (guillotine cuts) are allowed. Furthermore, the number of stages (set of cuts with the same orientation horizontal or vertical) in which a plate can be cut is also frequently limited to two or three.

An approach to obtain optimal solutions to different variants of the 2DCS problems, such as the existence of constraints on the type and number of cuts (two-stage and three-stage exact and non-exact) which is not based on column generation, is proposed.

The model is an extension of the one-cut model for the one-dimensional cutting stock problem proposed by Dyckhoff (1981). The approach is based on an integer programming model that can be solved by a general integer programming solver, taking advantage of the efficiency and robustness that (mixed-) integer programming solvers have achieved in recent years. For example, in the software used for the computational tests - CPLEX 11.0 (see ILOG, 2007) - several classes of valid inequalities and heuristics are implemented to improve the (lower and upper) bounds during the search of the branch-and-bound tree.

The proposed model for 2DCS uses the basic idea of the (one dimensional) Dyckhoff's one-cut model. Each decision variable is associated with cutting one item from a plate or from a part of a plate resulting from previous cuts (a residual plate). Constraints are related with the demands of the items and with the definition of the residual plates.

The model inherits the possibility of modeling the features already pointed out by Dyckhoff, as multiple types of pieces (plates) and the value of the pieces (plates) for future use. Furthermore, there is a feature of the proposed model, which is very relevant in practice but is not usually taken into account, which is modeling the length (or time) of the cuts needed to obtain all the final items.
Although pseudo-polynomial in size, the model optimized by a general-purpose integer programming solver (Cplex 11.0) solved a large number of "real-world" instances in a very reasonable amount of time. The computational

results confirm that the proposed model is sensitive to the number of item types of the instance and to the relation between the dimensions of the plate and the dimensions of the items (instances where each plate accommodates fewer items are easier for all solution approaches) but is not sensitive to the number of items.

For the 672 combinations of "real-world" instances and problems tested (168 instances for each problem), only in 29 occasions, corresponding to instances with a large number of item types, an optimal solution could not be found in two hours. In 21 of those occasions, the optimal solution is either the incumbent solution or uses only one plate less than the incumbent. For "real-world" instances, the behavior of models from the literature (which can also be directly solved by a general-purpose MIP solver) was clearly inferior to the one proposed, since those models are sensitive to the number of items of the instance and some instances have thousands of items. For instances adapted from the literature the behavior of the two models was similar.

However, the 2DCS is usually optimized individually and not studied as an integrated process in the industries. The cutting production planning is a repetitive process, occurring in the beginning of each time period, based on the costumers demands. Search for an optimal solution for a 2DCS in each period of the planning horizon, is different from a complete planning horizon evaluation. Considering production anticipation of some items or even the storage of *object remainders* (leftovers), with enough size to be cut in another period of the horizon planning, provides the change of a local improvement policy to a global optimum strategy. Thus, an interaction between time periods gives rise to an improvement of the production efficiency and a reduction of the costs.

Consequently, we also study the integration of the two-dimensional cutting stock problem and the lot-sizing problem (*2CS-LSP*). Given a demand of rectangular items during a given planning horizon, the problem is to determine the quantity of items that have to be cut, from the rectangular plates, in each time period of the planning horizon, such that the operation costs, the storage costs and wastage from the plates are minimized. Both 2CS and lot-sizing (*LS*) problems are NP-hard, hence its integration is also a very complex problem.

Integrating 2CSPand LS problems can be viewed as a balance between anticipate the production of some items, providing a better combination of the items in the larger rectangular plates, promoting the wastage minimization, and not anticipate the production of the items, fomenting the storage costs minimization. Solving these two problems separately may lead to non-optimal solutions to both 2CSP and LSP. An approach to obtain optimal solutions for the integrated 2CS-LSP, which is not based on the cutting patterns generation, is proposed. A mixed integer programming model is proposed, as an extension of the integer programming model that we propose to solve the 2CS problem individually.

The mixed integer programming model proposed allows solving optimally the integrated 2CS-LSP, by modeling the production anticipation of some items and also by storing object remainders, in the end of each time period of the planning horizon. The main purpose is to decide the quantity of items to cut in each time period, deciding if the cut of some items have to be or not anticipated, in a way that the costs of cutting, storage and also the number of initial plates used are minimized, respecting the demand of each item in each time period of the planning horizon. Computational tests are being conducted.

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Elsa Silva has graduated in Applied Mathematics from University of Minho in 2007. Currently, she is working in her Ph.D. thesis, her main research interests are exact and heuristic methods for two dimensional cutting stock and bin packing problems.

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